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An Efficient Reformulation of the Multiechelon Stochastic Inventory System with Uncertain Demands

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Significance

It is shown that by reformulating the three-stage multiechelon inventory system with specific exact linearizations, larger problems can be solved directly with mixed-integer linear programming (MILP) without decomposition. The new formulation is significantly smaller in the number of continuous variables and constraints. An MILP underestimation of the problem can be solved as part of a sequential piecewise approximation scheme to solve the problem within a desired optimality gap.

Keywords: optimization, mixed-integer linear programming, exact linearization, supply-chains

Introduction

In this article, we present an effective reformulation for the design of a multiechelon stochastic inventory system with uncertain customer demands. In Ref. 1, a three-echelon supply chain with inventories under uncertainty is presented. In that supply chain (see Figure 1), the location of the plants and the customer demand zones (CDZs) are known. Potential distribution centers (DCs) are given, and the objective is to decide which DCs to install to minimize total costs, which include transportation cost, installation cost, and inventory holding costs. The formulation also determines the service times for each DC, and what the size of the safety stock should be at all DCs and CDZs. This model uses single sourcing, which is often the case for supply chains in industrial gases or specialty chemicals. This means that all DCs are served by only one plant, and each CDZ is served by only one DC. The detailed model can be found in Ref. 1.

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In this short note, we first reformulate the mixed-integer nonlinear programming (MINLP) model presented in the literature,[1,2] using an discrete linearization scheme[3] leading to a model that is significantly smaller in size and tighter. This will greatly help any mixed-integer linear programming (MILP) solver and also increase the chance for obtaining a good solution quickly for the MINLP. We also present a successive piecewise linear approximation with which we can solve the model with a sufficiently small optimality gap using a normal MILP solver without the need of a global optimization solver.

Model formulation

In Ref. 1, the original MINLP model (P0) is stated as

$$\text{Min}: \sum_{j\in J} f_j Y_j + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} A_{ijk} X_{ij} Z_{jk} + \sum_{j\in J}\sum_{k\in K} B_{jk} Z_{jk} + \sum_{j\in J} q1_j \sqrt{N_j \sum_{k\in K} \sigma_k^2 Z_{jk}} + \sum_{k\in K} q2_k \sqrt{L_k} \quad (1)$$

s.t.

$$N_j \geq \sum_{i \in I} (SI_i + t1_{ij}) X_{ij} - S_j, \quad \forall j \tag{2}$$

$$L_k \geq \sum_{j \in J} (S_j + t2_{jk}) Z_{jk} - R_k, \quad \forall k \tag{3}$$

$$\sum_{i \in I} X_{ij} = Y_j, \quad \forall j \tag{4}$$

$$\sum_{j \in J} Z_{jk} = 1, \quad \forall k \tag{5}$$

$$Z_{jk} \leq Y_j, \quad \forall j, k \tag{6}$$

$$X_{ij}, Y_j, Z_{jk} \in \{0, 1\}, \quad \forall i, j, k \tag{7}$$

$$S_j \geq 0, \ N_j \geq 0, \quad \forall j \tag{8}$$

$$L_k \geq 0, \quad \forall k \tag{9}$$

with the following parameters (see Notation)

$$A_{ijk} = (c1_{ij}\chi + \theta 1_j t1_{ij}) \cdot \mu_k$$

$$B_{jk} = (g_j \chi + c2_{jk}\chi + \theta 2_k t2_{jk}) \cdot \mu_k$$

$$q1_j = \lambda 1_j \cdot h1_j$$

$$q2_k = \lambda 2_k \cdot h2_k \cdot \sigma_k$$

To decrease the number of nonlinear terms, the authors linearize in a similar way as proposed by Glover,[4] the bilinear terms in (P0) to obtain a new formulation, (P1), with fewer nonlinear constraints. In the objective function, Eq. 1, the product of two binaries $X_{ij}Z_{jk}$ is linearized by introducing continuous variables XZ_{ijk} and the following constraints

$$XZ_{ijk} \leq X_{ij}, \quad \forall i, j, k \tag{10}$$

$$XZ_{ijk} \leq Z_{jk}, \quad \forall i, j, k \tag{11}$$

$$XZ_{ijk} \geq X_{ij} + Z_{jk} - 1, \quad \forall i, j, k \tag{12}$$

$$XZ_{ijk} \geq 0, \quad \forall i, j, k \tag{13}$$

As the objective is to minimize the cost, Eqs. 10 and 11 can be removed without affecting the solution. In Eq. 3, the bilinear terms consisting of a continuous variable S_j times a binary variable Z_{jk} are linearized by introducing two new continuous variables, SZ_{jk} and $SZ1_{jk}$, as well as the following constraints

$$SZ_{jk} + SZ1_{jk} = S_j, \quad \forall j, k \tag{14}$$

$$SZ_{jk} \leq Z_{jk} \cdot S_j^U, \quad \forall j, k \tag{15}$$

$$SZ1_{jk} \leq (1 - Z_{jk}) \cdot S_j^U, \quad \forall j, k \tag{16}$$

$$SZ_{jk} \geq 0, \ SZ1_{jk} \geq 0 \quad \forall j, k \tag{17}$$

In a similar way, the bilinear terms between the continuous variables N_j and the binary variables Z_{jk}, in the objective function, are linearized as follows

$$NZ_{jk} + NZ1_{jk} = N_j, \quad \forall j, k \tag{18}$$

$$NZ_{jk} \leq Z_{jk} \cdot N_j^U, \quad \forall j, k \tag{19}$$

$$NZ1_{jk} \leq (1 - Z_{jk}) \cdot N_j^U, \quad \forall j, k \tag{20}$$

$$NZ_{jk} \geq 0, \ NZ1_{jk} \geq 0 \quad \forall j, k \tag{21}$$

Finally, the product under the square root term in the objective function is replaced with a variable NZV_j:

$$NZV_j = \sum_{k \in K} \sigma_k^2 \cdot NZ_{jk}, \quad \forall j \tag{22}$$

Although these linearization schemes are correct, they can be replaced with more effective formulations as will be shown next.

Alternative linearizations

In this section, we show how to linearize the model (P0) into a reformulated model (R1), which is more compact than the reformulated model (P1). First, the bilinear terms, $\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} A_{ijk} X_{ij} Z_{jk}$, in the objective function are similar to the objective in the quadratic assignment problem.[5] Therefore, we can use a similar approach, as in Ref. 3, to rearrange the variables in the objective function. As the parameter μ_k in $A_{ijk} = (c1_{ij}\chi + \theta 1_j t1_{ij}) \cdot \mu_k$ is only dependent on the index k, we can rewrite the objective function as follows

$$\sum_{j \in J} \left(\sum_{i \in I} A_{ij} X_{ij} \right) \sum_k Z_{jk} \mu_k \tag{23}$$

where

$$A_{ij} = (c1_{ij}\chi + \theta 1_j t1_{ij}) \tag{24}$$

is a new constant independent of k. According to Eq. 4, at most one of the X_{ij} variables in $\sum_i A_{ij} X_{ij}$ can be nonzero. Thus, this term can now be linearized using $|I| \cdot |J|$ new continuous variables, XZ_{ij}, instead of $|I| \cdot |J| \cdot |K|$ as in Eqs. 10–13. Hence, for the linearization, the objective function is written as

$$\sum_{j \in J} \sum_{i \in I} A_{ij} XZ_{ij} \tag{25}$$

and the new constraints (instead of Eqs. 10–13) as

$$XZ_{0j} + \sum_i XZ_{ij} = \sum_k \mu_k Z_{jk} \quad \forall j \tag{26}$$

$$XZ_{ij} \leq \sum_k \mu_k X_{ij} \quad \forall i, j \tag{27}$$

$$XZ_{0j} \leq \sum_k \mu_k (1 - Y_j) \quad \forall j \tag{28}$$

where the last variable Y_j comes from Eq. 4. If $Y_j = 0$, Eq. 6 will demand all Z_{jk} to be equal to zero as well. Therefore, the variable XZ_{0j} and Eq. 28 can be left out from the model, and the linearization can be written with only two constraints

$$\sum_i XZ_{ij} = \sum_k \mu_k Z_{jk} \quad \forall j \tag{29}$$

$$XZ_{ij} \leq \sum_k \mu_k X_{ij} \quad \forall i, j \tag{30}$$

As the objective is to minimize, and all variables are nonnegative, we can substitute the N_j in the objective function, with the right-hand side $\left(\sum_{i \in I} (SI_i + t1_{ij}) X_{ij} - S_j\right)$ of Eq. 2. Instead of having bilinear terms between the continuous variables N_j and the binary variables Z_{jk}, we can now linearize the expression below as follows

$$\left(\sum_{i \in I} (SI_i + t1_{ij}) X_{ij} - S_j\right) \cdot \left(\sum_{k \in K} Z_{jk} \sigma_k^2\right) \tag{31}$$

which can be written as

$$\left(\sum_{i \in I} (SI_i + t1_{ij}) X_{ij}\right) \cdot \left(\sum_{k \in K} Z_{jk} \sigma_k^2\right) - S_j \cdot \left(\sum_{k \in K} Z_{jk} \sigma_k^2\right) \tag{32}$$

Again, it can be noted that according to Eq. 4 at most one of the binary X_{ij} variables in the above equation can be nonzero. Therefore, the first part of this term can be linearized analogously as in Eqs. 29 and 30. Note that the variables are exactly the same in the bilinear terms, and, therefore, the same linearizations would work in both cases if it were not for the constants preceding the variables. However, we choose to write these linearizations with new continuous variables ZX_{ij} as well, as this formulation is tighter.

$$\sum_i ZX_{ij} = \sum_k \sigma_k^2 Z_{jk} \quad \forall j \tag{33}$$

$$ZX_{ij} \leq \sum_k \sigma_k^2 x_{ij} \quad \forall i, j \tag{34}$$

In the second half of Eq. 32, the bilinear terms in $S_j \cdot \left(\sum_{k \in K} Z_{jk} \sigma_k^2\right)$ are again exactly the same as the ones already linearized in Eqs. 14–16. Therefore, the bilinear terms are already defined earlier, and we can write the expression under the first square root term as

$$\sum_{i \in I} (SI_i + t1_{ij}) \cdot ZX_{ij} - \sum_{k \in K} \sigma_k^2 \cdot SZ_{jk} \tag{35}$$

Reformulated model

Based on the linearizations in the previous section, the reformulated nonlinear model (R1) is as follows

$$\text{Min} : \sum_{j \in J} f_j Y_j + \sum_{j \in J} \sum_{i \in I} A_{ij} XZ_{ij} + \sum_{j \in J} \sum_{k \in K} B_{jk} Z_{jk} + \sum_{j \in J} q1_j \sqrt{\sum_{i \in I} (SI_i + t1_{ij}) \cdot ZX_{ij} - \sum_{k \in K} \sigma_k^2 \cdot SZ_{jk}} + \sum_{k \in K} q2_k \sqrt{L_k} \tag{36}$$

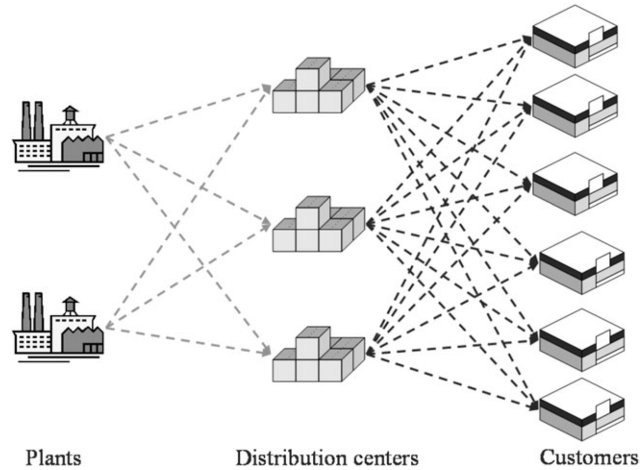

Figure 1. Network of multiple plants, DCs, and CDZs.

s.t.

$$L_k \geq \sum_{j \in J} SZ_{jk} + t2_{jk} \cdot Z_{jk} - R_k, \quad \forall k \tag{37}$$

$$\sum_{i \in I} X_{ij} = Y_j, \quad \forall j \tag{38}$$

$$\sum_{j \in J} Z_{jk} = 1, \quad \forall k \tag{39}$$

$$Z_{jk} \leq Y_j, \quad \forall j, k \tag{40}$$

$$\sum_i XZ_{ij} = \sum_k \mu_k Z_{jk} \quad \forall j \tag{41}$$

$$XZ_{ij} \leq \sum_k \mu_k X_{ij} \quad \forall i, j \tag{42}$$

$$\sum_i ZX_{ij} = \sum_k \sigma_k^2 Z_{jk} \quad \forall j \tag{43}$$

$$ZX_{ij} \leq \sum_k \sigma_k^2 X_{ij} \quad \forall i, j \tag{44}$$

$$SZ_{jk} + SZ1_{jk} = S_j, \quad \forall j, k \tag{45}$$

$$SZ_{jk} \leq Z_{jk} \cdot S_j^U, \quad \forall j, k \tag{46}$$

$$SZ1_{jk} \leq (1 - Z_{jk}) \cdot S_j^U, \quad \forall j, k \tag{47}$$

where all variables are ≥ 0. As shown later on in Table 3, this formulation is significantly smaller in size as well as tighter than (P1).

Concave nonlinear terms

In the same manner as in the original model,[1] the only remaining nonlinear terms in (R1) can be underestimated to obtain an MILP formulation (R2) of the model, which provides a good starting solution as well as a lower bound for (R1). As we use the same variables and constraints, every feasible solution of the MILP (R2) is also a feasible solution to the MINLP (R1). As we underestimate (R1), a valid lower bound for (R2) is also a lower bound for (R1). In the original paper,[1] (P1) is relaxed to obtain the MILP (P2), which is

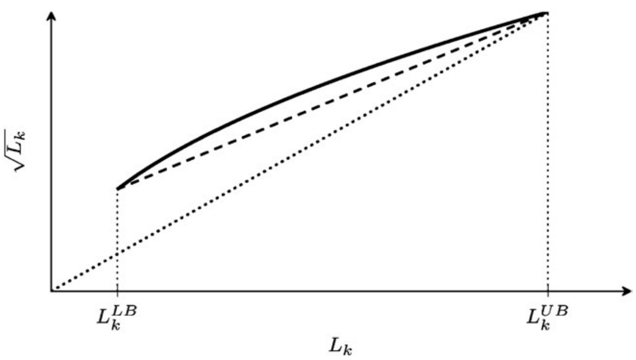

Figure 2. Underestimators with current lower bounds and with lower bound set to zero.

The black line is $\sqrt{L_k}$, the dashed line is Eq. 49, and the dotted line is the secant.

solved with CPLEX to obtain a good starting solution for the MINLP. You and Grossmann[1] underestimate the nonlinear square root terms $\sqrt{L_k}$ and $\sqrt{NZV_j}$ with their respective secants[6] $\frac{L_k}{\sqrt{L_k^{\mathrm{UB}}}}$ and $\frac{NZV_j}{\sqrt{NZV_j^{\mathrm{UB}}}}$. However, as one of the binary Z_{jk} variables in Eq. 3 is nonzero, the lower bound for L_k, when R_k is a parameter, becomes

$$L_k^{\mathrm{LB}} = \min_{j \in J}\{t2_{jk} - R_k\} \quad \forall k \tag{48}$$

To obtain a tighter underestimator, we use the following function, which is the secant in the feasible region of L_k

$$\frac{\left(\sqrt{L_k^{\mathrm{LB}}} - \sqrt{L_k^{\mathrm{UB}}}\right)}{\left(\left(L_k^{\mathrm{LB}}\right) - L_k^{\mathrm{UB}}\right)} L_k + \sqrt{L_k^{\mathrm{UB}}} - \frac{\left(\sqrt{L_k^{\mathrm{LB}}} - \sqrt{L_k^{\mathrm{UB}}}\right)}{\left(\left(L_k^{\mathrm{LB}}\right) - L_k^{\mathrm{UB}}\right)} L_k^{\mathrm{UB}} \tag{49}$$

In Figure 2, the difference is shown between underestimating the square root with the provided lower bounds vs. underestimating with the lower bound 0. When responsiveness R_k is defined as a variable in the model,[2] this is not the case.

Piecewise linear approximation

To tighten the underestimations for the convex square root terms, we use a sequential piecewise linear approximation approach. As the MILP (R2) formulation gives a relatively small gap, even on larger instances, we introduce a successive piecewise linear approximation scheme to obtain sufficiently small gaps. We use the δ-formulation[7] which is relatively tight.[8] As there are $|J| + |K|$ square root terms in the model, it is not a good idea to add too many intervals per

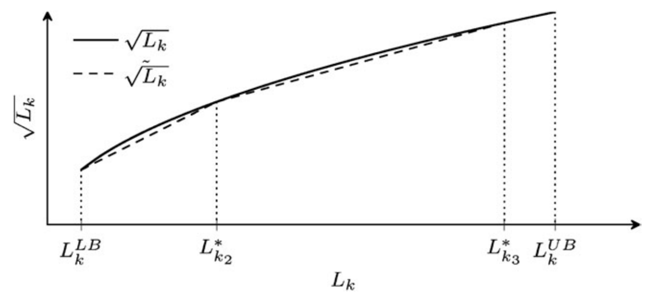

Figure 3. Intervals with the sequential approach after a few iterations.

square root term. Therefore, we solve the underestimating MILP several times, and only partition the terms that are greater than zero in the underestimation. In the equations below, we show the piecewise linearization used for the term $\sqrt{L_k}$. In the model, we also partition the second square root term analogously.

$$L_k = L_k^{\mathrm{LB}} + \sum_{n=1}^{N} \delta_n \tag{50}$$

$$\widetilde{\sqrt{L_k}} = \sqrt{L_k^{\mathrm{LB}}} + \sum_{n=1}^{N} \frac{\sqrt{L_{k_{n+1}}^*} - \sqrt{L_{k_n}^*}}{L_{k_{n+1}}^* - L_{k_n}^*} \delta_n \tag{51}$$

$$\left(L_{k_{n+1}}^* - L_{k_n}^*\right) w_n \le \delta_n \le \left(L_{k_{n+1}}^* - L_{k_n}^*\right) w_{n-1}, \quad n = 2, \ldots, N-1 \tag{52}$$

$$\left(L_{k_2}^* - L_{k_1}^*\right) w_1 \le \delta_1 \le L_{k_2}^* - L_{k_1}^* \tag{53}$$

$$\delta_N \le \left(L_k^{\mathrm{UB}} - L_{k_N}^*\right) w_{N-1} \tag{54}$$

$$w_n \in \{0, 1\}, \quad n = 1, \ldots, N-1 \tag{55}$$

In the sequential approach, we start by solving the MILP model (R2), then we add a grid point for each square root term that is larger than its lower bound. The found MILP solution will, therefore, be exact for the MINLP, but now with a higher optimal value due to the added grid points. Then, we solve the model again with the new grid points, and if we find a better solution, we again add grid points at the new solution as shown in Figure 3. Therefore, N, which is the number of grid points, will be different for all square root terms. The grid intervals will depend on the solutions found and will in general not be equally spaced. Most of the

Table 1. Sizes of the Old and the Reformulated Models

			(P2)			(R2)		
$\vert I\vert$	$\vert J\vert$	$\vert K\vert$	Bin. Vars.	Cont. Vars.	Const.	Bin. Vars.	Cont. Vars.	Const.
2	20	20	460	2480	5300	460	940	1800
5	30	50	1680	13,640	33,190	1680	3410	6520
10	50	100	5550	70,250	185,350	5550	11,200	21,400
20	50	100	6050	120,250	335,350	6050	12,200	22,400
3	50	150	7700	52,800	120,450	7700	15,650	30,900
15	100	200	21,600	300,300	1,040,700	21,600	43,400	83,800

Table 2. Comparison of the Tightness in the Root Node of the Relaxed MILPs (P2) and (R2)

$\|I\|$	$\|J\|$	$\|K\|$	LB (P2)	LB (R2)
2	20	20	1,024,834	1,584,557
5	30	50	2,087,756	3,142,002
10	50	100	3,550,280	5,070,126
20	50	100	3,234,617	4,531,680
3	50	150	4,779,460	7,122,785
15	100	200	6,468,441	8,735,968

terms will stay at zero, and, therefore, no new grid points or variables will be needed for those terms. After no improvement is found, the final MILP with the added grid points is solved to a predefined optimality gap. The lower bound for the piecewise approximation is also a valid lower bound for (R1), while every feasible solution for (R2) is also a feasible solution for (R1). As the piecewise underestimator is exact in the grid points, this approach can also be used to close the gap completely. This is a similar approach as the branch-and-refine method found in the literature.[9,10]

Computational results

In this section, we present computational results on the same size instances as in Ref. 1. All computations in this article are conducted on an Asus UX31E ultra book with a 2.8-GHz quad-core Intel processor and 4 Gb of RAM. As a MILP solver, we used CPLEX 12.3 with the default parameters. All constant values in the model are generated randomly with uniform distribution within predefined values. These values can be found in Notation. To be able to reproduce the runs, the default random seed in GAMS 23.7.3 was used. The difficulties of these problems are strongly dependent on the values assigned to the different costs.

In Table 1, the model sizes of (P2) and (R2) are compared. As can be seen, the difference is very large, especially for the larger instances. The root node values of the relaxed LP for the different formulations are shown in Table 2. As can be seen from Tables 1 and 2, (R2) is both smaller in size and significantly tighter than (P2). In fact, the first MILP instance with two suppliers, 20 DCs and 20 CDZs, is solved already at the root node.

In Table 3, the computational comparisons of the reformulated MILP (R2) and the original formulation (P2) are presented. As can be observed, (P2) can only be solved for the smallest instance within the time limit of 10 min. On the other hand, for the new formulation (R2), all underestimating MILP instances are solved in a few seconds with 0% optimality gap. This comparison is made with the same secant function for the square root terms in both cases (i.e., zero lower bound in Eq. 49). If the square root term in (R2) is relaxed with the tighter underestimator shown in Eq. 49, then the optimal values of the MILP will be higher and, therefore, closer to the optimal value of (R1). However, the solution times for (R2) would still be as fast as in Table 3.

Table 4. Gap Between the Optimal Solution of (R2) and the Calculated Feasible MINLP Solution

$\|I\|$	$\|J\|$	$\|K\|$	MILP (R2)	MINLP (R1)	Gap (%)	Time (s)
2	20	20	1,708,930	1,783,540	4.18	0.09
5	30	50	3,538,980	3,770,404	6.14	0.67
10	50	100	5,862,307	6,215,596	5.68	4.74
20	50	100	5,294,980	5,396,136	1.87	6.86
3	50	150	8,161,606	8,479,354	3.75	1.65
15	100	200	10,318,630	10,727,860	3.81	28.2

Table 4 shows the global optimum of the MILP (R2), when underestimating the square root terms with the tighter lower bound in Eq. 49 and the corresponding calculated feasible solution of the problem when evaluating the square root terms. Note that the optimal solutions for (R2) are higher in Table 4 (i.e., tighter lower bound for (R1)) than in Table 3 for the same problems, while the solution times are roughly the same. When the underestimating MILP (R2) is solved, every feasible solution for (R2) is also a feasible solution for (R1) (with a different objective value), and every valid lower bound of (R2) is also a lower bound for (R1). The gap showed in Table 4 is calculated from these bounds.

In Table 5, the solution times are shown for the sequential piecewise approach presented in the last section. Even the larger instances can be solved within reasonable computational time. However, the solution times for different instances with the same size can be completely different. This formulation is strongly dependent on the values of the parameters. This is also why the solution time is longer for the third instance in Table 5 than for the fourth, although the fourth instance is larger in size. The gap is calculated from the best (highest) lower bound and the lowest enumerated MINLP solution found during the solution process.

Conclusions

In this short note, we have showed that by reformulating the three-stage multiechelon inventory system with specific exact linearizations, we can solve larger problems directly with an MILP solver without the need of decomposing the problem. The new formulation is significantly smaller in

Table 3. Comparison of the Performance of the Underestimating MILPs (P2) and (R2), with a Time Limit of 600 s

			(P2)				(R2)			
$\|I\|$	$\|J\|$	$\|K\|$	UB	LB	Gap (%)	Time (s)	UB	LB	Gap (%)	Time (s)
2	20	20	1,584,557	1,584,557	0.00	1.27	1,584,557	1,584,557	0.00	0.09
5	30	50	3,248,076	2,988,151	7.80	600	3,169,880	3,169,880	0.00	0.58
10	50	100	6,893,324	3,569,929	48.20	600	5,173,888	5,173,888	0.00	4.18
20	50	100	5,969,190	3,245,514	45.63	600	4,602,706	4,602,706	0.00	6.15
3	50	150	10,852,632	5,009,423	53.84	600	7,151,942	7,151,942	0.00	2.92
15	100	200	54,755,897	6,407,108	88.30	600	8,816,596	8,816,596	0.00	23.4

Table 5. Solutions for Some Instances with the Sequential Piecewise Approach

$\|I\|$	$\|J\|$	$\|K\|$	Solution	LB	Gap (%)	Time (s)
2	20	20	1,776,969	1,770,058	0.39	4
5	30	50	3,728,020	3,708,151	0.53	57
10	50	100	6,182,142	6,120,321	0.99	8326
20	50	100	5,396,136	5,358,578	0.70	87
3	50	150	8,479,354	8,426,424	0.62	81
15	100	200	10,704,022	10,597,052	0.99	10,202

size, both in the number of continuous variables and in the number of constraints. An MILP underestimation of the problem can be solved very quickly to obtain a good feasible solution for the MINLP. The article shows a simple sequential piecewise approximation scheme that can be used to solve the problem within a desired optimality gap.

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Notation

Models

(P0) = original MINLP
(P1) = original reformulated MINLP
(P2) = original underestimating MILP
(R1) = reformulated (P1)
(R2) = reformulated (P2)

Sets

I = set of suppliers
J = set of candidate DC locations
K = set of CDZs

Parameters

Below are the input parameters for the instances. All parameters are generated randomly with a uniform distribution. The data files used are available from the authors, upon request.

$c1_{ij} = t1_{ij} \times U[0.05, 0.1]$, unit transportation cost between supplier and DC
$c2_{jk} = t2_{jk} \times U[0.05, 0.1]$, unit transportation cost from DC to CDZ
$f_j = U[150,000, 160,000]$, fixed cost for installing a DC at location j
$g_j = U[0.01, 0.1]$, annual variable cost coefficient for installing DC at location j
$h1_j = U[0.1, 1]$, unit inventory holding cost at DC
$h2_k = U[0.1, 1]$, unit inventory holding cost at CDZ
$R_k = 0$, maximum guaranteed service time to customers at CDZ k
$SI_i = U[1, 5]$(integers), guaranteed service time of plant i
$t1_{ij} = U[1, 7]$(integers), order processing time of DC j if it is served by supplier i
$t2_{jk} = U[1, 3]$(integers), order processing time of CDZ k if it is served by DC j
$\mu_k = U[75, 150]$, daily mean demand at CDZ k
$\sigma_k^2 = U[0, 50]$, daily variance of demand at CDZ k
$\chi = 365$, days per year
$\theta1_{ij} = U[0.1, 1]$, annual unit cost of pipeline inventory from plant i to DC j
$\theta2_{jk} = U[0.1, 1]$, annual unit cost of pipeline inventory from DC j to CDZ k
$\lambda1_j = 1.96$, safety stock factor of DC j
$\lambda2_k = 1.96$, safety stock factor of CDZ k

Binary variables

$X_{ij} = 1$ if DC j is served by supplier i
$Y_j = 1$ if a DC is installed at location j
$Z_{jk} = 1$ if the CDZ is served by DC j

Continuous variables

L_k = net lead time of CDZ k
N_j = net lead time of DC j
S_j = guaranteed service time of DC j to the CDZs

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